

JANUARY 2004

**EXAMINATION
MATHEMATICS**

Paper 02 – General Proficiency

2 hours 40 minutes

05 JANUARY 2004 (a.m.)

INSTRUCTIONS TO CANDIDATES

1. Answer ALL questions in Section 1, and any TWO from Section II.
2. Write your answers in the booklet provided.
3. All working must be shown clearly.
4. A list of formulae is provided on page 2 of this booklet.

Examination Materials

Electronic calculator (non-programmable)

Geometry set

Mathematical tables (provided)

Graph paper (provided)

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

LIST OF FORMULAE

Volume of a prism $V = Ah$ where A is the area of a cross-section and h is the perpendicular length.

Volume of a right pyramid $V = \frac{1}{3}Ah$ where A is the area of the base and h is the perpendicular height.

Circumference $C = 2\pi r$ where r is the radius of the circle.

Area of a circle $A = \pi r^2$ where r is the radius of the circle.

Area of trapezium $A = \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the perpendicular distance between the parallel sides.

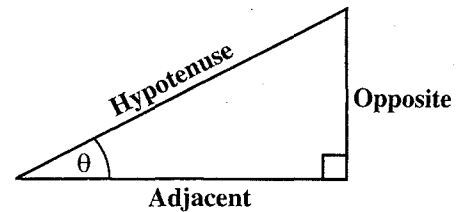
Roots of quadratic equations If $ax^2 + bx + c = 0$,
then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



Area of triangle

Area of $\Delta = \frac{1}{2}bh$ where b is the length of the base and h is the perpendicular height

$$\text{Area of } \Delta ABC = \frac{1}{2}ab \sin C$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

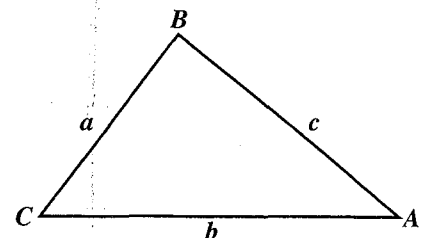
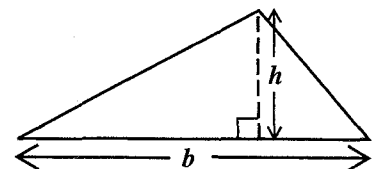
$$\text{where } s = \frac{a + b + c}{2}$$

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$



SECTION I

Answer ALL the questions in this section.

All working must be clearly shown.

1. (a) Find the exact value of

$$\frac{2\frac{1}{5} - 1\frac{2}{3}}{\frac{4}{7}}$$

giving the answer as a fraction in its LOWEST terms. (3 marks)

- (b) The cash price of a dining room suite with a table and six identical chairs is \$880. If the price of the table is \$250, what is the price of EACH chair? (2 marks)
- (c) The dining room suite may be bought on hire purchase for a deposit of \$216 plus monthly payments of \$35 for a period of two years. Calculate

- (i) the total hire purchase price of the suite
- (ii) the extra cost of buying on hire purchase as a percentage of the cash price. (5 marks)

Total 10 marks

2. (a) If $p = 5$, $q = 0$ and $r = -3$, calculate the value of

(i) $4p - qr$

(ii) $2r^3$. (3 marks)

- (b) The temperature, C , in degrees Celsius, is calculated using the formula

$$C = \frac{5}{9} (F - 32)$$

where F is the temperature in degrees Fahrenheit.

- (i) Make F the subject of the formula.
- (ii) The temperature in London is 15°C . Use the formula derived in (i) above to convert this temperature to degrees Fahrenheit. (5 marks)
- (c) Solve the following pair of simultaneous equations:

$$2x + 3y = 18$$

$$x + 5y = 23.$$

(4 marks)

Total 12 marks

3. (a) The following information is given:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$P = \{1, 2, 5, 10\}$$

$$Q = \{2, 3, 5, 8, 9\}$$

P and Q are subsets of U , the Universal set.

- (i) Draw a Venn diagram to represent the information above.

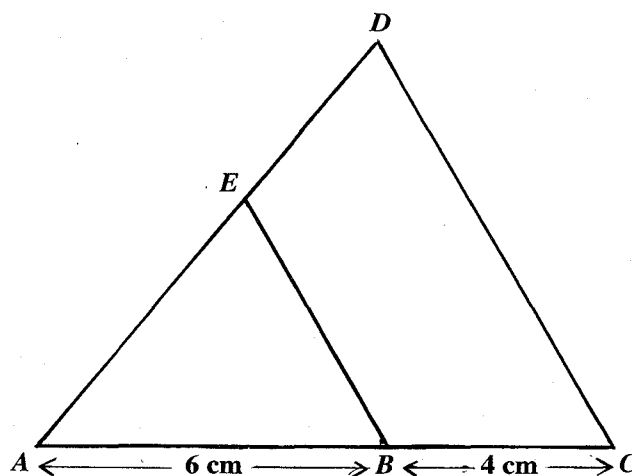
- (ii) List, using set notation, the members of the set

a) $P \cap Q$

b) $(P \cup Q)'$

(6 marks)

- (b) In the diagram below, **not drawn to scale**, triangle ACD is an enlargement of triangle ABE , with A as the centre of enlargement, $AB = 6$ cm and $BC = 4$ cm.



Calculate

- (i) the scale factor of the enlargement which maps triangle ABE onto triangle ACD .
- (ii) the area of triangle ACD , in cm^2 , given that the area of triangle ABE is 18 cm^2 .

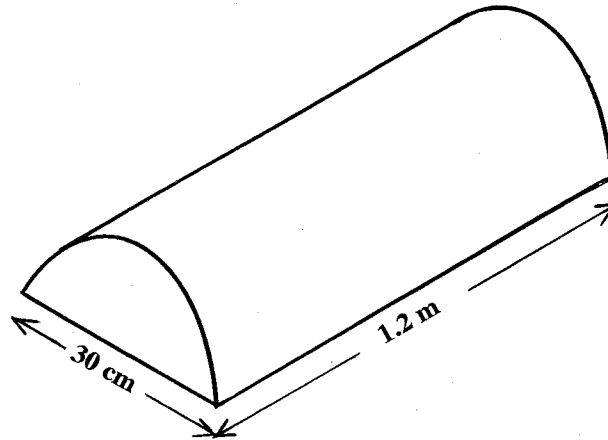
(4 marks)

Total 10 marks

4. (a) Mrs Haynes took her class on a tour of the island. The table below shows the time the bus arrived at two places of interest along the route.

	Place along the route	
	South Point	Bird Park
Arrival time	09:38	11:16
Departure time	09:40	11:20

- (i) How long did it take the bus to travel from South Point to Bird Park?
- (ii) South Point is 80 km away from Bird Park. Calculate the speed of the bus, in km/h, along this section of the route. **(6 marks)**
- (b) The diagram below, **not drawn to scale**, shows a block of wood in the shape of a semi-circular prism. The cross section of the prism is a semi-circle with diameter 30 cm. The length of the prism is 1.2 metres.



Use $\pi = 3.14$.

Calculate, giving your answer to 3 significant figures

- (i) the area, in cm^2 , of the cross section
- (ii) the volume, in cm^3 , of the prism. **(5 marks)**

Total 11 marks

5. (a) Given that $f(x) = 3x + 4$ and $g(x) = \sqrt{x}$, calculate

(i) $g(25)$

(ii) $gf(15)$.

(3 marks)

(b) (i) Given that $y = x^2 - 3x$, copy and complete the table below.

x	-2	-1	0	1	2	3	4	5
y		4	0	-2		0	4	10

(ii) Using a scale of 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the y -axis, draw the graph of $y = x^2 - 3x$ for $-2 \leq x \leq 5$.

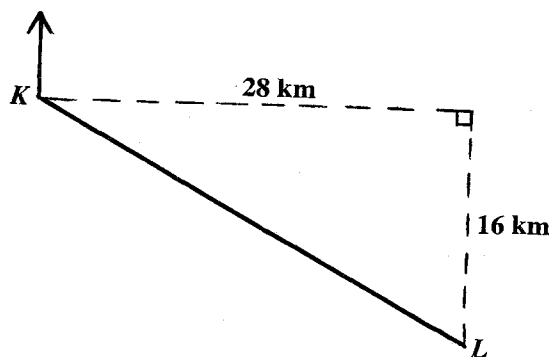
(iii) On the same axes as (ii) above, draw the line $y = x$ for $-2 \leq x \leq 5$.

(iv) Use your graphs to determine the solution of the equation $x^2 - 3x = x$.

(10 marks)

Total 13 marks

6. (a) The diagram below, **not drawn to scale**, shows the journey of a ship which sailed from Port K to Port L . Port L is located 28 km East and 16 km South of Port K .



Calculate, giving your answer correct to 3 significant figures

(i) the distance KL

(ii) the bearing of Port L from Port K .

(6 marks)

An answer sheet is provided for Question 6 (b)

(b) The diagram on the answer sheet shows triangle XYZ and its image $X'Y'Z'$ under a reflection, R .

(i) Draw the mirror line and label it M .

(ii) Write down the y intercept of the mirror line M .

(iii) Determine the equation of the mirror line M .

(6 marks)

Total 12 marks

7. **An answer sheet is provided for this question.**

The table below shows

- (i) a grouped frequency distribution of the marks scored by 40 students on a test, and
- (ii) an incomplete column of cumulative frequencies.

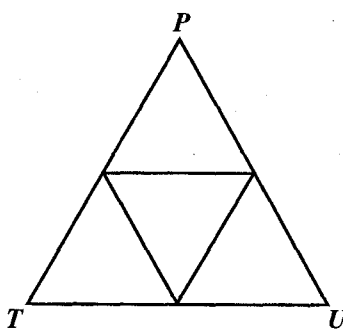
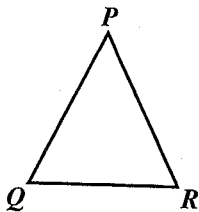
Interval	Frequency	Cumulative Frequency
1 – 5	4	4
6 – 10	5	9
11 – 15	9	—
16 – 20	14	—
21 – 25	5	—
26 – 30	3	—

- (a) Calculate the mean of the frequency distribution assuming that each mark within each interval has a value equal to the midpoint of the interval. **(4 marks)**
- (b) Copy and complete the cumulative frequency table. **(2 marks)**
- (c) Using the cumulative frequency curve on the answer sheet provided, find
 - (i) the median mark scored
 - (ii) the number of students who scored no more than 12 marks
 - (iii) the number of students who scored more than 8 but less than 18 marks
 - (iv) the probability that a student chosen at random would score no more than 24 marks. **(6 marks)**

Draw lines on the answer sheet to show how all values were obtained.

Total 12 marks

8. In the diagram below, **not drawn to scale**, the equilateral triangle PQR , of side one unit, is the basic triangle from which larger equilateral triangles may be formed, as seen in triangle PTU .



- (a) Using ruler and compasses only, construct triangle PVY where each side is of length 6 cm. **(3 marks)**
- (b) Draw lines on your diagram to sub-divide triangle PVY into nine identical equilateral triangles. **(2 marks)**
- (c) The table below shows how the number of unit lengths in each side of a figure is related to the number of basic triangles in that figure. Copy and complete the table.

Number of unit lengths in each side of figure	1	2		8
Number of basic triangles in that figure	1	4	9	

(2 marks)

- (d) In the basic triangle PQR , the length of each side is 2 cm. Calculate the area of the basic triangle in cm^2 . **(3 marks)**

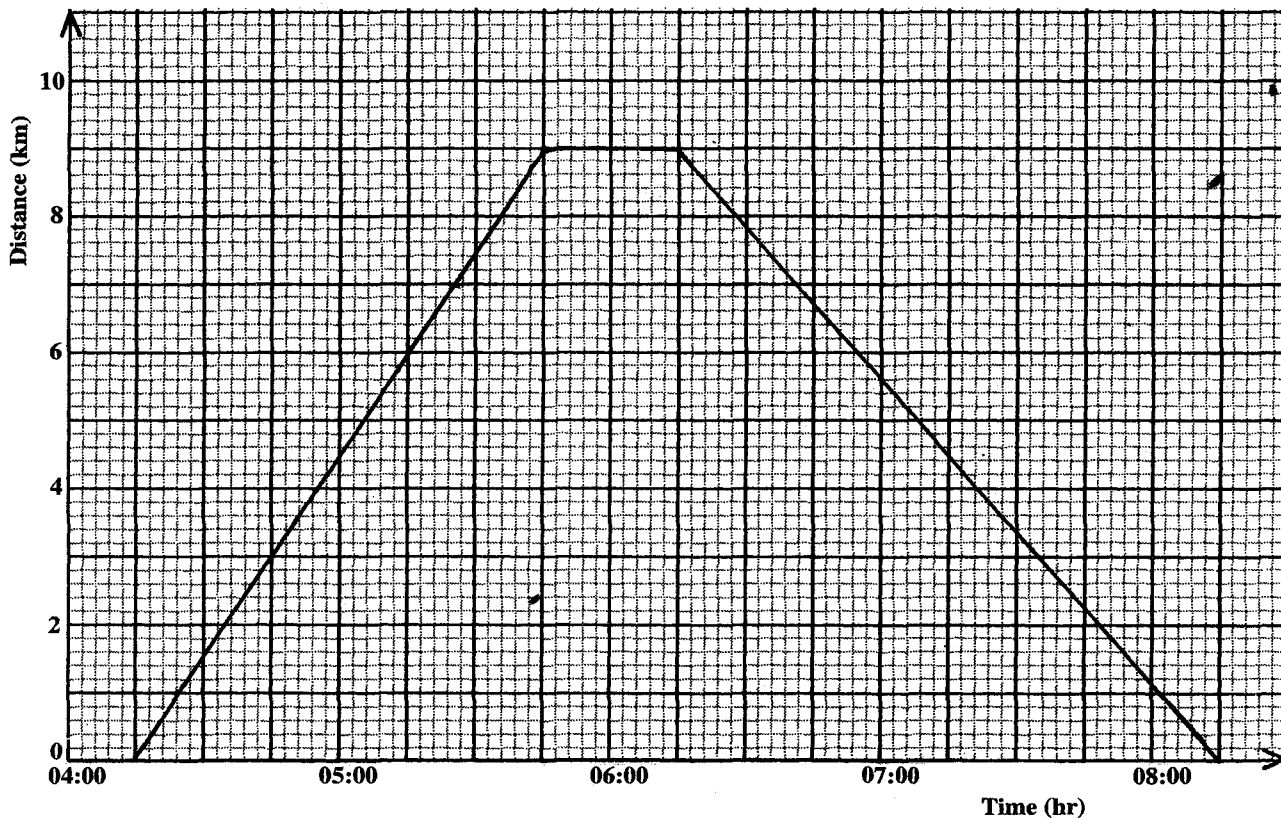
Total 10 marks

SECTION II

Answer TWO questions in this section.

RELATIONS, FUNCTIONS AND GRAPHS

9. The graph below represents the journey of an athlete running from the training camp to the park and back to the camp.



- (a) Using the graph, determine
- (i) the time at which the athlete left the training camp
 - (ii) the distance from the camp to the park
 - (iii) the length of time he spent at the park
 - (iv) the speed of the athlete on his way to the park, in km/h.

(5 marks)

- (b) A cyclist leaves the camp at 6:00 hrs and rides to the park at 10 km/h. Determine
- (i) the time at which the cyclist meets the athlete
 - (ii) the distance from the park to where the cyclist and the athlete meet.

(3 marks)

- (c) (i) Write $f(x) = 4x^2 - 7x + 3$ in the form $f(x) = a(x + b)^2 + c$, where a , b and c are constants.
- (ii) State
- a) the minimum value of $f(x)$
- b) the value of x for which the minimum occurs. (7 marks)

Total 15 marks

10. (a) Mrs Singh owns a clothing store. She buys x dresses and y shirts from a factory at wholesale prices.
- (i) For a wholesale purchase, she must buy AT LEAST 15 dresses and AT LEAST 20 shirts. Write TWO inequalities to represent this information.
- (ii) Mrs Singh has \$2 400 to spend on the dresses and shirts. Each dress costs \$40 and each shirt costs \$30. Write an inequality to represent this information. (4 marks)
- (b) Using a scale of 1 cm to represent 5 units on each axis, draw a graph of the THREE inequalities and label the region, R, which satisfies ALL of the inequalities. (7 marks)
- (c) Mrs Singh sells the dresses and shirts in her store. She makes a profit of \$25 on each dress and \$6 on each shirt.
- (i) Write an expression for the profit, P .
- (ii) Determine the number of dresses and shirts that Mrs Singh should buy to make the maximum profit.
- (iii) Calculate the maximum profit. (4 marks)

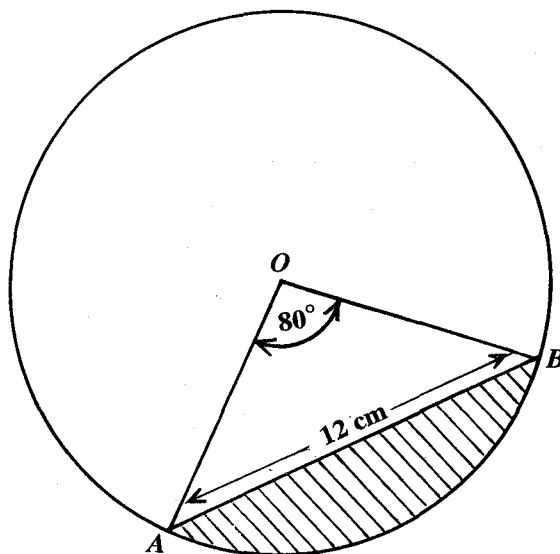
Total 15 marks

GEOMETRY AND TRIGONOMETRY

11. (a) Use $\pi = 3.14$ and the radius of the Earth, R , is 6370 km.

Two airports S and T are located at $(30^\circ\text{N}, 25^\circ\text{W})$ and $(30^\circ\text{N}, 20^\circ\text{E})$ respectively. Calculate, **to the nearest km**, the SHORTEST distance between S and T , along the parallel of latitude. **(4 marks)**

- (b) In the diagram below, **not drawn to scale**, O is the centre of the circle. Angle $AOB = 80^\circ$ and $AB = 12$ cm.



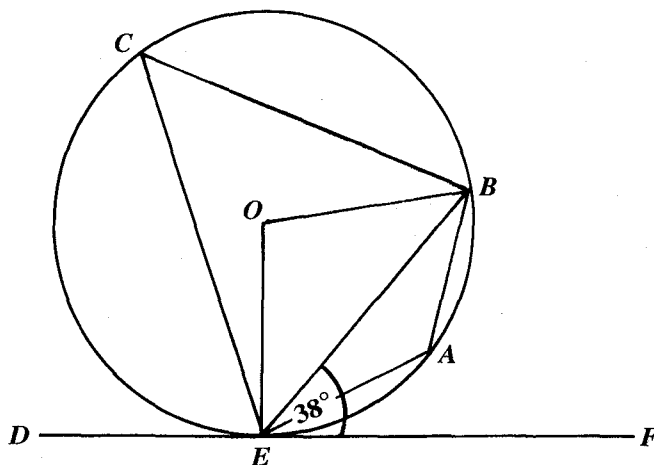
Calculate, giving your answer correct to 2 decimal places

- (i) the radius of the circle
- (ii) the area of the minor sector AOB
- (iii) the area of the shaded region.

(11 marks)

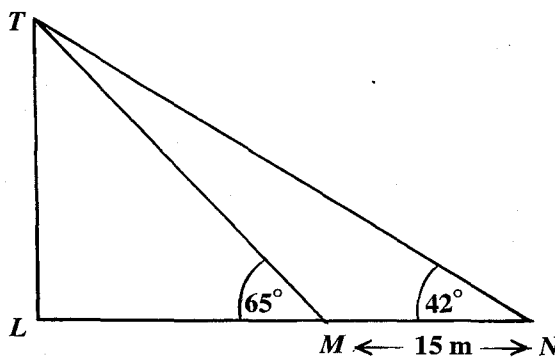
Total 15 marks

12. (a) In the diagram below, **not drawn to scale**, DEF is a tangent to the circle $ABCE$, with centre O . The measure of angle BEF is 38° .



Calculate, giving reasons for your answers, the size of EACH of the following angles:

- (i) $\angle BEO$
 - (ii) $\angle BCE$
 - (iii) $\angle BAE$
 - (iv) $\angle BOE$
- (8 marks)
- (b) In the diagram below, **not drawn to scale**, TL is a vertical tower, and LMN is a straight line on a horizontal plane. The angles of elevation of T , the top of the tower from the points M and N are 65° and 42° respectively. The distance of M from N is 15 m.



- (i) Using triangle MNT , show that the length of MT to the nearest metre, is 26 m.
 - (ii) Hence, calculate the height of the tower, TL .
- (7 marks)

Total 15 marks

VECTORS AND MATRICES

13. ABC is a triangle. The coordinates of the points A , B and C are $(2,2)$, $(6,2)$ and $(4,6)$ respectively.

- (a) Write in the form $\begin{bmatrix} x \\ y \end{bmatrix}$, the position vectors of A , B and C . (2 marks)
- (b) Write in the form $\begin{bmatrix} x \\ y \end{bmatrix}$, the vectors
- (i) \overrightarrow{AB}
- (ii) \overrightarrow{AC}
- (iii) \overrightarrow{BC} . (4 marks)
- (c) Show by a vector method that triangle ABC is an isosceles triangle. (5 marks)
- (d) Determine the position vector of the point D , such that BD is parallel to AC and $ABDC$ forms a parallelogram. (4 marks)

Total 15 marks

14. (a) The matrix $C = \begin{bmatrix} 6 & 2 \\ 5 & p \end{bmatrix}$ is a singular matrix. Calculate the value of p . (2 marks)

- (b) (i) Given that $3x + 5y = 19$
 $2x + 4y = 14$

express the pair of equations in the form $AX = B$, where A , X and B are matrices.

- (ii) Determine the inverse of matrix A .
- (iii) Evaluate: $A^{-1}B$. (5 marks)

(c) V is the transformation represented by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and

W is the transformation represented by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

- (i) Describe FULLY the transformation, V .
- (ii) Describe FULLY the transformation, W .
- (iii) Write the single matrix $\begin{bmatrix} p & r \\ q & p \end{bmatrix}$, which represents the combined transformation, V followed by W .
- (iv) Calculate the image of the point $(6,-4)$ under the combined transformation in (c) (iii) above. (8 marks)

Total 15 marks

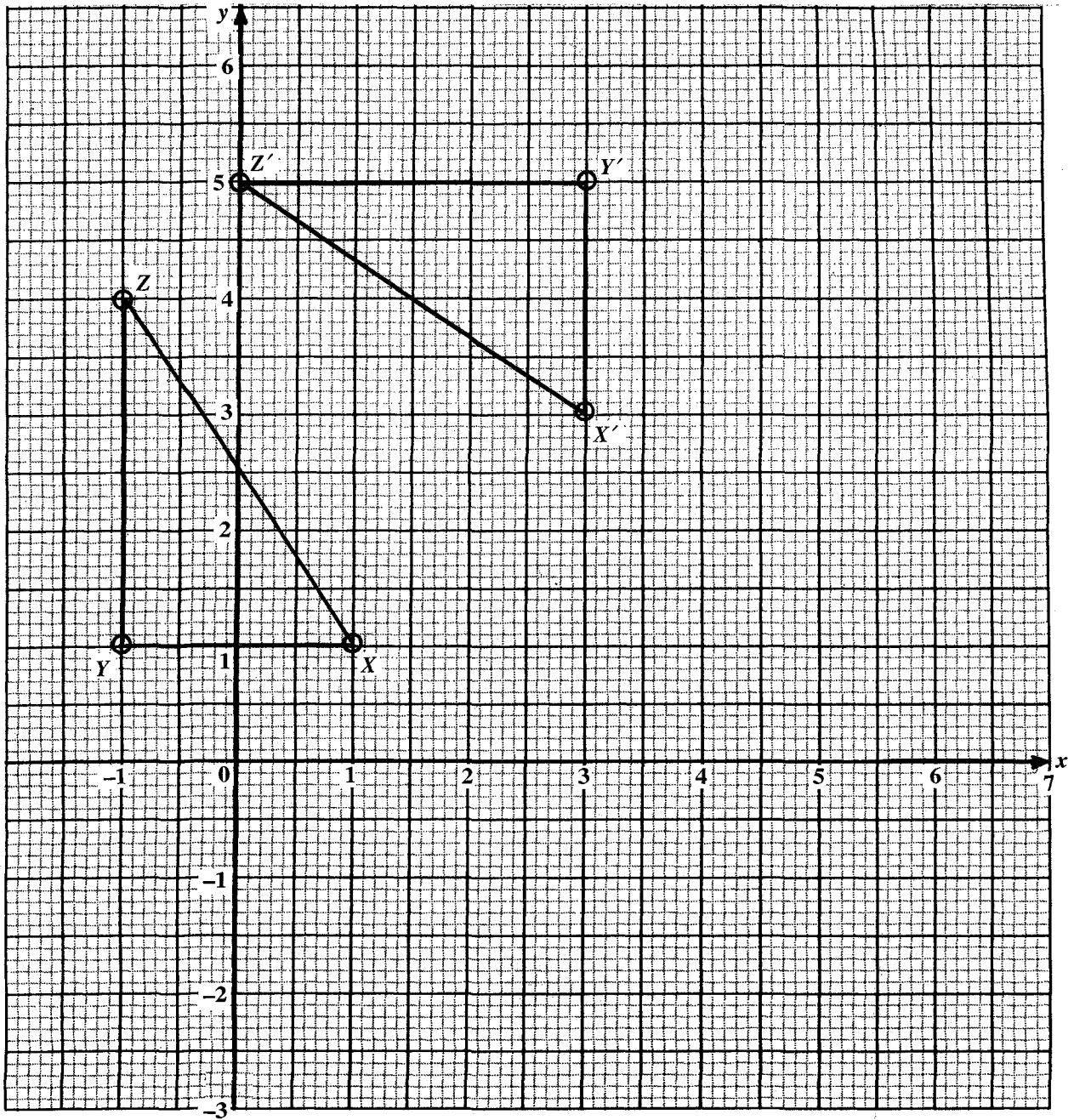
END OF TEST

EXAMINATION
MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 6 (b)

Candidate Number



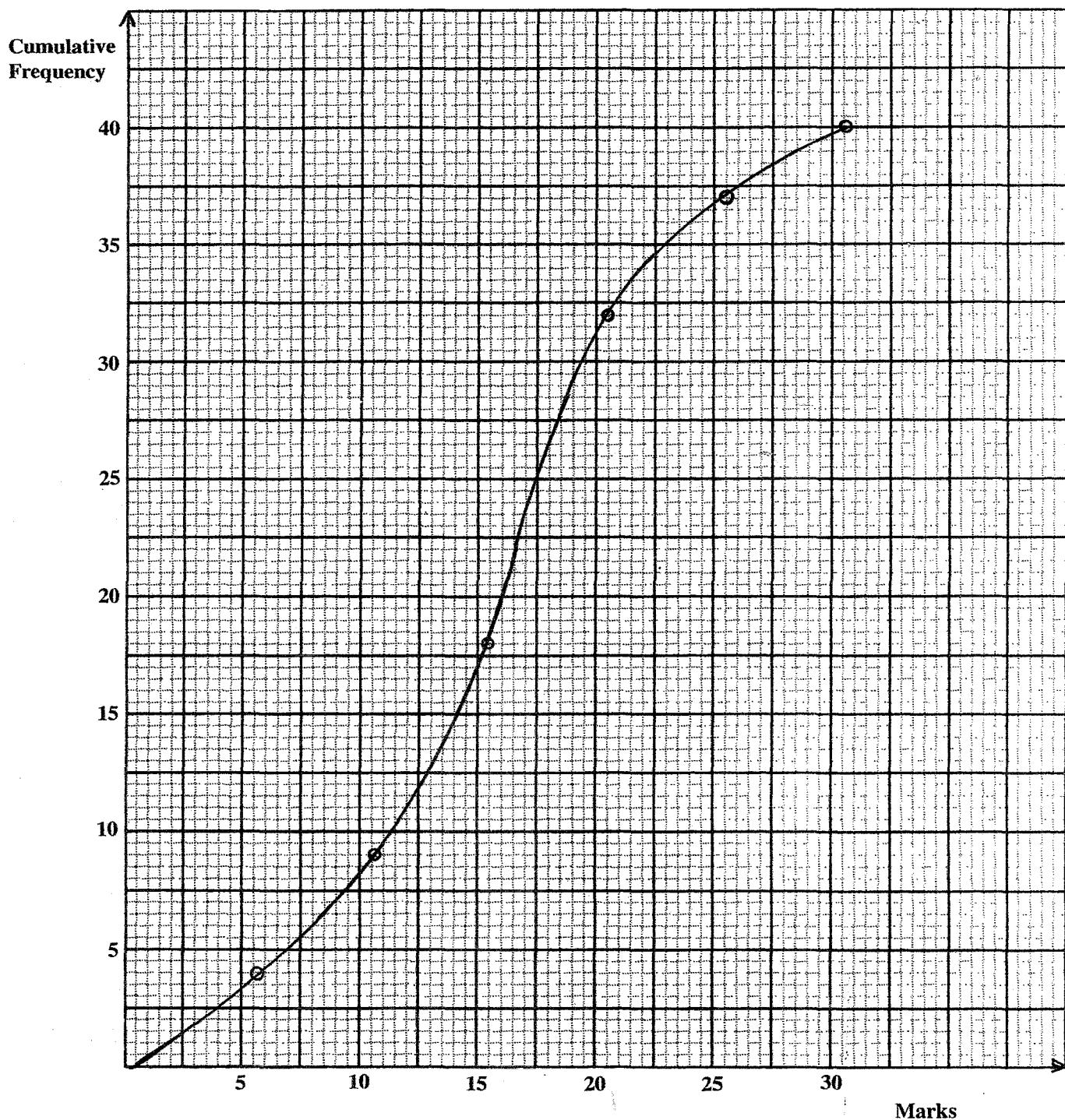
ATTACH THIS ANSWER SHEET TO YOUR BOOKLET

EXAMINATION
MATHEMATICS

Paper 02 – General Proficiency

Answer Sheet for Question 7 (c)

Candidate Number



ATTACH THIS ANSWER SHEET TO YOUR BOOKLET